ICS 2022 Problem Sheet #7

**Problem 7.1:** quine-McCluskey algorithm

A Boolean function *F* is defined using the following sum of minterms:

F (A, B, C, D, E) =m0+m2+m4+m6+m9+m10+m13+m14+m15+m16+m17+m21+m26+m28+m30+m31

1. Calculate the prime implicants of *F* .

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| minterm | pattern | used | minterms | pattern | used | minterms | pattern | used |
| m0 | 00000 | \* | m0,2 | 000-0 | \* | m0,2,4,6 | 00--0 |  |
| m0,4 | 00-00 | \* |  |  |  |
| m0,16 | -0000 |  |  |  |  |
| m2 | 00010 | \* | m2,6 | 00-10 | \* |  |  |  |
| m2,10 | 0-010 | \* | m2,10,6,14 | 0--10 |  |
| m4 | 00100 | \* | m4,6 | 001-0 | \* |  |  |  |
| m16 | 10000 | \* | m16,17 | 1000- |  |  |  |  |
| m6 | 00110 | \* | m6,14 | 0-110 | \* |  |  |  |
| m9 | 01001 | \* | m9,13 | 01-01 |  |  |  |  |
| m10 | 01010 | \* | m10,26 | -1010 | \* | m10,14,26,30 | -1-10 |  |
| m10,14 | 01-10 | \* |  |  |  |
| m17 | 10001 | \* | m17,21 | 10-01 |  |  |  |  |
| m13 | 01101 | \* | m13,15 | 011-1 |  |  |  |  |
| m14 | 01110 | \* | m14,15 | 0111- | \* | m14,15,30,31 | -111- |  |
| m14,30 | -1110 | \* |  |  |  |
| m21 | 10101 | \* |  |  |  |  |  |  |
| m26 | 11010 | \* | m26,30 | 11-10 | \* |  |  |  |
| m28 | 11100 | \* | m28,30 | 111-0 |  |  |  |  |
| m15 | 01111 | \* | m15,31 | -1111 | \* |  |  |  |
| m30 | 11110 | \* | m30,31 | 1111- | \* |  |  |  |
| m31 | 11111 | \* |  |  |  |  |  |  |

Prime implicants: m0,16 m9,13 m16,17 m17,21 m13,15 m28,30 m0,2,4,6 m2,10,6,14 m14,15,30,31 m10,14,26,30

F = m0,16 + m9,13 + m16,17+ m17,21 + m13,15 + m28,30 + m0,2,4,6 + m2,10,6,14 + m14,15,30,31 + m10,14,26,30

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | m0 | m2 | m4 | m6 | m9 | m10 | m13 | m14 | m15 | m16 | m17 | m21 | m26 | m28 | m30 | m31 |
| m0,16 | \* |  |  |  |  |  |  |  |  | \* |  |  |  |  |  |  |
| m16,17 |  |  |  |  |  |  |  |  |  | \* | \* |  |  |  |  |  |
| m9,13 |  |  |  |  | + |  | + |  |  |  |  |  |  |  |  |  |
| m17,21 |  |  |  |  |  |  |  |  |  |  | + | + |  |  |  |  |
| m13,15 |  |  |  |  |  |  | \* |  | \* |  |  |  |  |  |  |  |
| m28,30 |  |  |  |  |  |  |  |  |  |  |  |  |  | + | + |  |
| m0,2,4,6 | + | + | + | + |  |  |  |  |  |  |  |  |  |  |  |  |
| m2,10,6,14 |  | \* |  | \* |  | \* |  | \* |  |  |  |  |  |  |  |  |
| m10,14,26,30 |  |  |  |  |  | + |  | + |  |  |  |  | + |  | + |  |
| m14,15,30,31 |  |  |  |  |  |  |  | + | + |  |  |  |  |  | + | + |
|  | + | + | + | + | + | + | + | + | + |  | + | + | + | + | + | + |

1. Construct the prime implicant chart and identify the essential prime implicants.

Essential prime implicants: m9,13 m17,21 m28,30 m0,2,4,6 m14,15,30,31 m10,14,26,30

F1 = m0,16 + m9,13 + m17,21 + m28,30 + m0,2,4,6 + m14,15,30,31 + m10,14,26,30 cost(F1) = 4 \* 3 + 3 \* 2 + 6 = 24

F2 = m16,17 + m9,13 + m17,21 + m28,30 + m0,2,4,6 + m14,15,30,31 + m10,14,26,30 cost(F2) = 4 \* 3 + 3 \* 2 + 6 = 24

1. Write out all minimal boolean expressions defining *F* using the mathematical logic notation.
2. The minimal boolean expressions defining *F* using the mathematical logic notation with prime implicants:

F = m0,16 + m9,13 + m16,17+ m17,21 + m13,15 + m28,30 + m0,2,4,6 + m2,10,6,14 + m14,15,30,31 + m10,14,26,30

= B’C’D’E’ + A’BC’D + AB’C’D’ + AB’C’E + A’BCE + ABCE’ + A’B’E’ + BDE’ + BCD + BDE’=

= BCDE ABCD) (BCD) BCE) (ABCE) (ABE) (BDE) (BCD) (BDE).

1. The minimal boolean expressions defining *F* using the mathematical logic notation with essential prime implicants:

F1 = m0,16 + m9,13 + m17,21 + m28,30 + m0,2,4,6 + m14,15,30,31 + m10,14,26,3

= B’C’D’E’ + A’BC’D + A’BC’E +ABCE’ + A’B’E’ + BCD + BDE’=

= BCDE) ABCD) BCE) (ABCE) (ABE) (BCD) (BDE).

F1 = m16,17 + m9,13 + m17,21 + m28,30 + m0,2,4,6 + m14,15,30,31 + m10,14,26,3

= AB’C’D’ + A’BC’D + A’BC’E +ABCE’ + A’B’E’ + BCD + BDE’=

= (BCD)) ABCD) BCE) (ABCE) (ABE) (BCD) (BDE).